Representing, Electing and Ranking

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Mechanisms -- "instruments or processes, physical or mental, by which something is done or comes into being" -- have been invented across the ages to regulate human activities of all kinds: markets and their prices for economic decisions, electoral systems for political decisions, ad hoc rules of every imaginable variety for competitions (sports, musical, wine, beauty or other, or for whom is to have the priority for a kidney transplant).

What mechanisms should be used? The answers depend on the contexts and the mathematical properties they elicit. Practice demands that they be simple, clear and understood by all: else they would be suspect, hence rejected. The focus here will be on the mechanisms for elections (with asides concerning their applications elsewhere).

Lecture 1. How to apportion fairly: Aristotle, the Talmud, the U.S Congress and kidney transplants.

The story begins with Aristotle and the Talmud, and goes on to the disputes over the meaning of "Representatives ... shall be apportioned among the several States ...according to their respective numbers ..." among Washington, Hamilton, Jefferson, J.Q. Adams, Webster and later Morse, von Neumann and Cornell's Willcox.

Underlying concepts of fairness determine the mechanism that should be used to apportion representatives. They show why Daniel Webster and after him Walter Willcox had the right idea and the phalanx of the American mathematical elite of the day -- which successfully argued for the method used since 1940 -- erred. It also explains why mechanisms for allocating kidneys to awaiting patients can be unfair.

Lecture 2. How to elect and to rank: overcoming the paradoxes of social choice, electing a President of France and ranking wines.

Ramon Llull in 1299 and Nicolas Cusanus in 1433 proposed mechanisms for electing and ranking that continue to be championed to the present day, though they are attributed, respectively, to the Marquis de Condorcet (1789), and the Chevalier de Borda (1770). They established a way of seeing the problem and modelled it in terms that remains the central paradigm of the theory of social choice to this day. As a consequence, Arrow's "impossibility theorem" showing that Condorcet's famous paradox cannot be escaped, has been interpreted as meaning that there is no satisfactory mechanism for either designating a winner or determining a ranking among competitors.

A new theory -- whose antecedents may be traced to the Marquis de Laplace and Sir Francis Galton -- sees the problem and models it differently. It suggests that the usual interpretation is wrong: Arrow's theorem means that there is no satisfactory mechanism unless there is a common language of measuring the qualities of candidates (or wines) or the performances of competitors. But when a common language exists -- and practice shows they do exist -- then one mechanism should be used: the majority judgement. This is explained in the context of the 2007 French presidential elections (and, time permitting, in the context of judging wines).

Lecture 3. How to eliminate gerrymandering: a new approach to representation, its realization in Zürich and its application to the U.S. Congress.

Elbridge Gerry -- Governor of Massachusetts (1810-12), Vice President of the United States (1813-14) -- gave his name to the dubious "practice of dividing a geographical area into electoral districts, often of highly irregular shape, to give one political party an unfair advantage." This ancient fine art has become a new 21st century technology that, some say, determines electoral outcomes more than do the voters. And yet it redistricts to meet the one criterion championed by the Supreme Court -- "a good-faith effort to achieve precise mathematical equality": every one of Pennsylvania's 19 districts has a census population of either 646,371 or 646,372, and every one of Texas' 32 has a population of 651,619 or 651,620. Justice John Harlan was unusually prescient in his dissenting opinion of 1969 (Wells v. Rockefeller 394 U.S. 542): "The fact of the matter is that the rule of absolute equality is perfectly compatible with 'gerrymandering' of the worst sort. A computer may grind out district lines which can totally frustrate the popular will ... The legislature must do more than satisfy one man, one vote; it must create a structure which will in fact as well as
theory be responsive to the sentiments of the community ... Even more than in the past, district lines are likely to be
drawn to maximize the political advantage of the party temporarily dominant in public affairs."

A new mechanism (extending that of lecture 1) provides the structure sought by John Harlan. It is today the law of the
land for the legislative elections of the city and the canton of Zürich, due to a suit of a citizen who complained that his
constitutional right to an equal vote was violated by the old system. Adopted for U.S. congressional elections, that
mechanism would eliminate the possibility of gerrymandering for political advantage.

References:

Lecture 1

1. Michel L. Balinski and H. Peyton Young, *Fair Representation: Meeting the Ideal of One Man, One Vote*, Yale


Lecture 2

Academy of Sciences, USA* 104 (2007) 8720-8725.

413-419.


4. See also the web site: [www.ceco.polytechnique.fr/jugement-majoritaire.html](http://www.ceco.polytechnique.fr/jugement-majoritaire.html)

Lecture 3


43-53.

3. Michel Balinski, “Fair majority voting (or how to eliminate gerrymandering),” to appear in *American Mathematical

Wissenschaft*, April 2007, 76-80.

H. Styan (Eds.), *Festschrift for Tarmo Pukkila*, Department of Mathematics, Statistics and Philosophy, University of